TRANSLATIONAL MOTION OF ELASTIC OBJECTS

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Abstract: *Using control of translational motion of an elastic object with linear-viscous resistance due to the selection of the type and parameters of control the absolute quiescence of the object at the end of movement is provided.*
Key words*: elastic object, translational optimal movement, linear-viscous resistance, inverse task of dynamics.*

Introduction.

There are studies on the control of oscillations of linear and nonlinear mechanical systems in absolute motion [1, 2]. Works [3, 4] are devoted to optimal control of translational and rotational movements of the elastic systems with finite or infinite number of degrees of freedom. There is a need to use such special movement controls, in which fluctuations of transported objects are significantly reduced or completely eliminated, i.e. in an acceptable minimum possible time of translational motion the relative or absolute quiescence at the end of the movement is achieved [5].

The purpose of the research is the accounting of the linear-viscous resistance in relative motion with optimal translational motion of an elastic object. Here, the optimal (purposeful) movement means the existence of a functional-criterion that receives a stationary value in the actual movement [3, 4].

Optimal control of the translational motion with $U_e(t) = a\sin^3(pt)$**.** It should be noted that control $U_e(t) = a\sin^3(pt)$ is the solution of the differential equation

$$\frac{d^4U_e}{dt^4} + 10p^2\frac{d^2U_e}{dt^2} + 9p^4U_e = 0$$

considering boundary conditions

$$t = 0, \quad U_e(0) = 0, \quad \dot{U}_e(0) = 0; \qquad t = T/4, \quad U_e(T/4) = a, \quad \dot{U}_e(T/4) = 0.$$

If we double integrate control $U_e(t) = a\sin^3(pt)$ considering additional boundary conditions

$$t = 0, \quad S_e(0) = 0, \quad V_e(0) = 0; \quad t = T, \quad S_e(T) = L,$$

and after determining the arbitrary constants the expressions for the displacement, velocity and acceleration of translational motion come:

$$U_e(t) = \frac{3\pi L}{T^2}\sin^3\left(\frac{2\pi}{T}t\right), \qquad V_e(t) = \frac{L}{2T}\left(\cos^3\left(\frac{2\pi}{T}t\right) - 3\cos\left(\frac{2\pi}{T}t\right) + 2\right),$$

$$S_e(t) = \frac{L}{12\pi T}\left(12\pi t - T\sin^3\left(\frac{2\pi}{T}t\right) - 6T\sin\left(\frac{2\pi}{T}t\right)\right).$$

At L = 1 m; T = 2 s graphics $U_e(t)$, $V_e(t)$, $S_e(t)$ are depicted in figure 1.

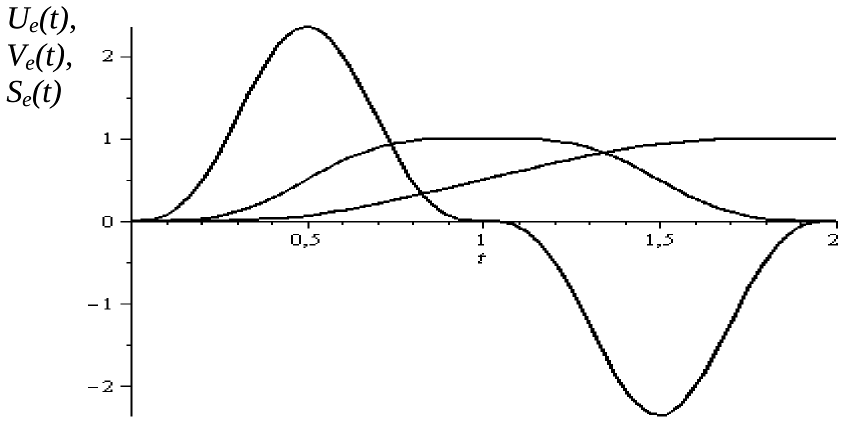

Fig. 1. Translational motion graphics for $U_e(t) = a\sin^3(pt)$

The graphics show that at time $t = T$ displacement is equal $S_e(T) = L$, velocity and acceleration respectively are $V_e(T) = 0$, $U_e(T) = 0$, that is the object is brought into the translational quiescence. As previously, this control is applicable to the displacement of an elastic object, according to the theory of moments, taking into account reasonable motion time T. Here at $n = 0$ the solution of the equation

$$\frac{d^2x_r}{dt^2} + 2n\frac{dx_r}{dt} + k^2x_r = -a\sin^3(pt)$$

is following:

$$x_r(t) = -\left[\frac{6ap^3}{A} + \frac{a}{B}\left(3k^2 - 27p^2\right)\right]\sin pt + \left(p^2 - k^2\right)\sin 3pt,$$

where

$$A = \left(k^4 - 10k^2T^2\pi^2 + 144\pi^4\right)k;$$

$$B = 4k^4T^4 - 160k^2T^2\pi^2 + 570\pi^4.$$

From moment ratios $x_r(T) = 0,\quad \dot{x}_r(T) = 0$ at $p = 2\pi/T$ and $k = 2\pi n_1/T$ it is possible to find motion time T, at which relative quiescence of the elastic system is achieved.

After transformation from moment ratios $(x_r(T) = 0,\ \dot{x}_r(T) = 0)$ we obtain transcendental equations (graphics are depicted in figure 2)

$$S_1(n) = \sin(2\pi n_1) = 0, \quad S_2(n) = \cos(2\pi n_1) - 1 = 0,$$

which have following acceptable (for this task) conjoint roots $n_1 = 2, 3, 4,\ldots$ For example, for $n_1 = 4$, $T = \dfrac{2\pi n_1}{k}$, $k = 4\pi$, $a = \dfrac{3L\pi}{T^2}$ relative motion graphics $x_r(t)$ and $\dot{x}_r(t)$, depicted in figure 3, show that relative quiescence comes at moment of time $t = T$; in combination with the translational quiescence the required absolute quiescence of the moving elastic object is achieved.

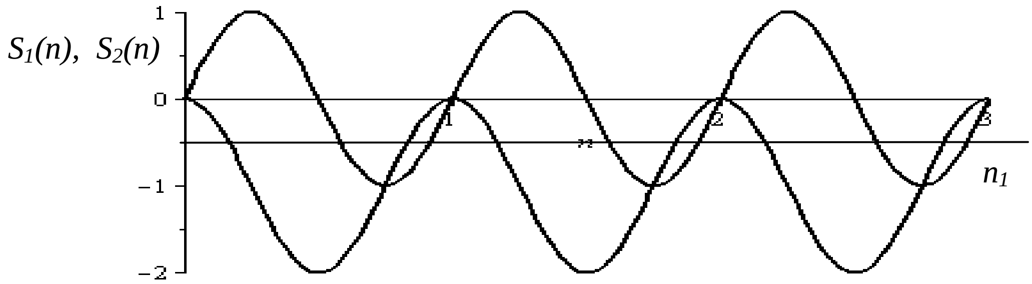

Fig. 2. Moment ratios graphics

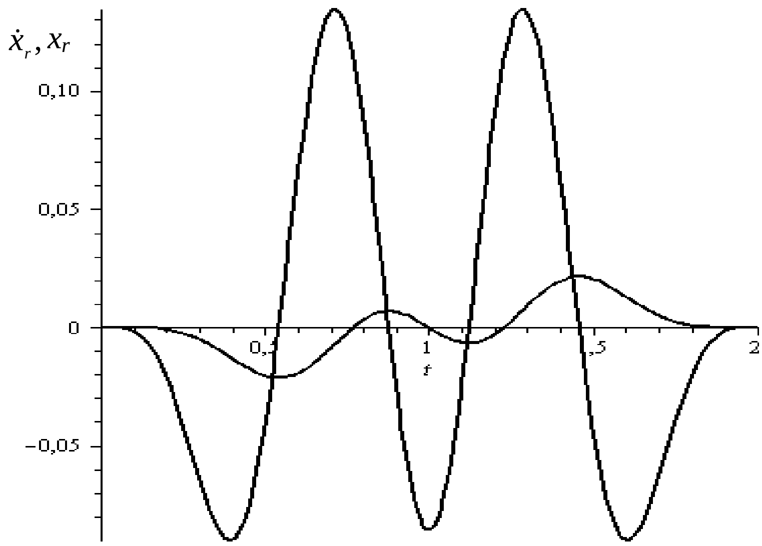

Fig. 3. Relative motion graphics using $U_e(t) = a\sin^3(pt)$ $(n = 0)$

Conclusions. 1. The procedure for finding conjoint roots of transcendental equations that represent the moment ratios for the system with linear-viscous resistance can be eliminated by using the algorithm of the inverse task of dynamics, that is, finding the control to implement the "ideal" motion. **2**. Examined approach can be applied to a wide class of skew-symmetric controls, optimality of which is justified by using the reverse principle, thus analytical control function corresponds to the Euler equation of the functional – optimality criterion, which takes stationary value in time interval of motion of the object.

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